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Quantifying the SIGMA X crossing benefit

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Related analysis:

Sofianos, **Dark Pools and Algorithmic Trading**, *Algorithmic Trading, A Buy-Side Handbook (2nd Edition), The Trade*, July 07

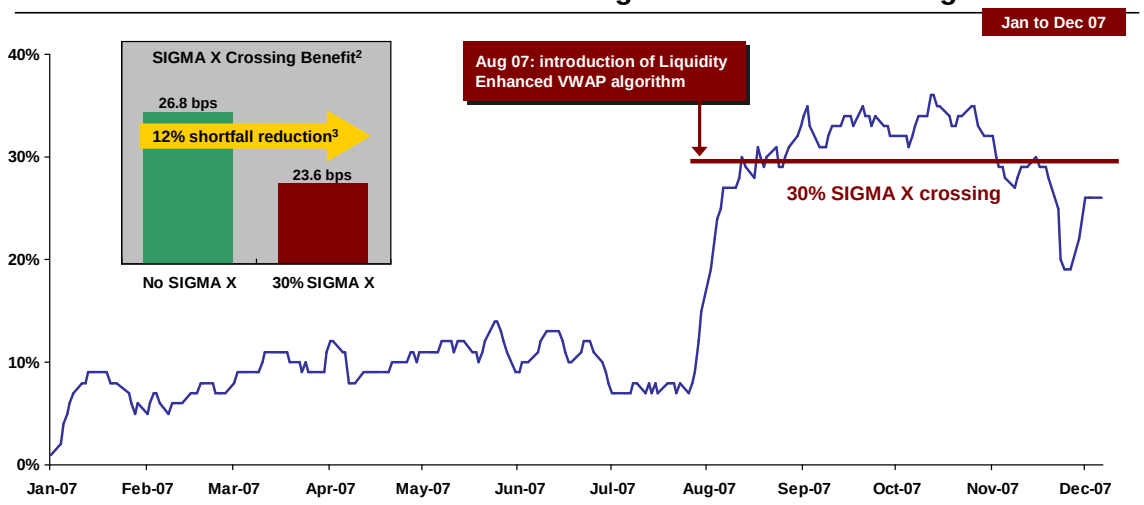
Abrokwah & Sofianos, **Shortfall Surprises**, *Journal of Trading*, Summer 07

In the United States, Goldman Sachs algorithms cross an increasing amount of orders within SIGMA X, Goldman Sachs' internal pool of non-displayed liquidity.[1] In this issue of *Street Smart*, we focus on the US VWAP algorithm and quantify the reduction in execution shortfall resulting from SIGMA X crossing.[2] In our analysis, we use multivariate regressions to control for the various factors that influence execution shortfall. **Based on our analysis, we find that, for the average VWAP order in our sample, SIGMA X crossing reduces execution shortfall (relative to arrival price) by 12 percent, from 26.8 to 23.6 bps**. We also find that the reduction in shortfall caused by SIGMA X:

- Is larger for orders in mid-cap and small-cap stocks
- Increases with execution horizon and order size
- Is most likely caused by reduced information leakage

SIGMA X crosses 140 million shares daily and one billion shares flow through the system.[3] Our analysis suggests that order exposure to this large pool of non-displayed liquidity improves execution quality. We next discuss our findings in more detail.

Exhibit 1: The SIGMA X benefit and crossing rates for the VWAP algorithm[1]



1. Our sample begins January 26, 07 and we report rolling five-day average crossing rates.
2. Shortfall regression simulations, VWAP algorithm market orders greater than 1,000 shares and execution half-life greater than 30 minutes, August to December 07.
3. We measure shortfall relative to the midquote at order arrival (execution shortfall)

Source: Goldman Sachs Equity Execution Strategies

The Goldman Sachs VWAP algorithm and SIGMA X crossing

VWAP algorithms divide the execution horizon into brief time buckets then slice "parent" orders into small "child" orders and allocate them across the buckets in proportion to expected volume. The Goldman Sachs VWAP algorithm exposes these child orders to SIGMA X liquidity, without displaying them to the market. If no counterparty is found within SIGMA X, the child orders are routed to the public market for execution. All Goldman Sachs algorithms take advantage of SIGMA X liquidity, but in this issue of *Street Smart* we focus on the VWAP algorithm.

Exhibit 1 shows the SIGMA X crossing rates for VWAP child orders. Until August 07, the average crossing rate was 11 percent. **In August, Goldman Sachs introduced its new Liquidity**

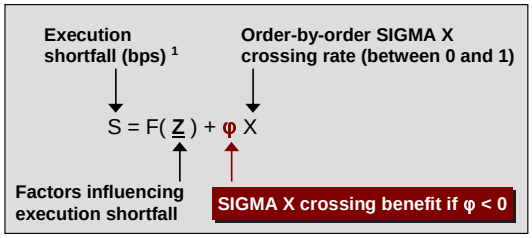
Enhanced VWAP algorithm and SIGMA X crossing jumped to 30 percent. The Liquidity Enhanced VWAP algorithm looks to execute targeted volumes by placing non-displayed child orders in SIGMA X at the mid-point prior to going out into the displayed market.

The SIGMA X crossing of child orders may reduce execution shortfall on the parent order in three ways:

- SIGMA X may cross child orders inside the spread (e.g., at midquote)
- SIGMA X does not display child orders to the market, so SIGMA X crossing reduces information leakage[4]
- SIGMA X crossing accelerates the execution of the parent order (to the extent allowed by the algorithm execution schedule) and since the typical order has short-term alpha this reduces shortfall

Our objective is to quantify by how much the SIGMA X crossing of child orders reduces execution shortfall on the parent order. **We call this reduction in shortfall the "SIGMA X crossing benefit."**

Exhibit 2. Controlling for other factors



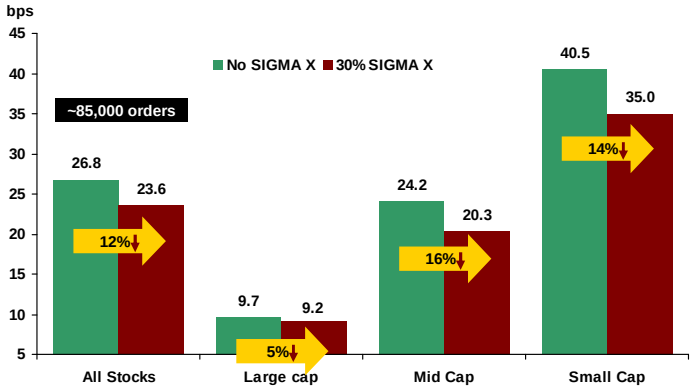
1. Buy orders: execution price minus midquote at order arrival as percent of midquote.

Source: Goldman Sachs Equity Execution Strategies

A framework for quantifying the SIGMA X crossing benefit

To quantify the SIGMA X crossing benefit, we use a sample of client VWAP algorithm orders executed by Goldman Sachs between August and December 07. Our sample consists of VWAP *parent* orders and excludes orders where the client specified a limit price.[5] For each parent order in our sample, we calculate its actual execution shortfall and SIGMA X crossing rate. Some orders have zero SIGMA X crossing, some execute 100 percent within SIGMA X and some execute partially within SIGMA X.

Exhibit 3. Quantifying the SIGMA X crossing benefit[1]



1. Regression simulations, VWAP algorithm market orders greater than 1,000 shares and execution half-life greater than 30 minutes, August to December 07.

Source: Goldman Sachs Equity Execution Strategies

To isolate the SIGMA X crossing benefit we must control for other factors that may affect execution shortfall. We do this by estimating regressions (Exhibit 2) of execution shortfall (S) on the various factors (**Z**) that may affect execution shortfall and on the order-by-order SIGMA X crossing rate (X). The various factors **Z** include the participation rate, stock volatility, quoted spreads and execution horizon alpha. The Appendix discusses our regressions in more detail.

In Exhibit 2, φ measures the effect of SIGMA X on execution shortfall, holding all else equal. Negative φ means SIGMA X crossing reduces execution shortfall. In our preferred regression φ is -10.6 bps and statistically significant (see Appendix). **This value of φ suggests that 30 percent SIGMA X crossing, the average in our sample, will reduce execution shortfall by 3.2 bps.** In the next section we discuss our findings in more detail.

SIGMA X crossing and the reduction in execution shortfall

The sample for our preferred regressions consists of 85,000 VWAP parent orders that are greater than 1,000 shares and with execution horizon greater than 30 minutes.[6] Exhibit 3 summarizes our main findings by simulating our preferred regressions under two scenarios:[7]

- Assuming zero SIGMA X crossing the execution shortfall for the average order in our sample is **26.8 bps**
- **Assuming 30 percent SIGMA X crossing (and holding all other factors constant) execution shortfall is reduced by 12 percent to 23.6 bps**

In Exhibit 3 we also split our sample into three stock capitalization buckets and re-estimated our preferred regression for each bucket.[8] We find that for mid-cap and small-cap stocks, 30

percent SIGMA X crossing reduces execution shortfall by **15 percent**. For large-cap stocks the corresponding reduction is only **5 percent**. **Most of the SIGMA X benefit, therefore, is in mid-cap and small-cap stocks**.

Our findings are robust across different regression specifications, different cuts of the order data and different sub-periods. Exhibit 4, for example, summarizes a set of twenty regressions where we split our sample into five buckets based on each order's execution horizon and stock capitalization. Looking at the first row (all stocks), we find a strong SIGMA X benefit across all execution horizon buckets.[9] Moreover, the estimates show that **the SIGMA X benefit is larger the longer the execution horizon**. For example, the SIGMA X benefit is **2.3 bps** for execution horizons greater than 15 minutes and **5.8 bps** for execution horizons greater than 3 hours.

Exhibit 4 also confirms our finding that the SIGMA X benefit is stronger for mid-cap and small cap-stocks. For orders with execution horizon more than three hours, for example, SIGMA X crossing has no effect on orders in large-cap stocks but reduces execution shortfall by **6.5 bps** for mid-cap and by **8.5 bps** for small-cap stocks.

Exhibit 4. SIGMA X benefit by execution horizon

Assuming 30 percent SIGMA X crossing

	Reduction in execution shortfall (bps)[1]				
	Execution half-life[2] greater than:				
	15 min	30 min	1 hour	2 hours	3 hours
All stocks	**-2.3**	**-3.2**	**-3.6**	**-3.7**	**-5.8**
t-statistic	*-9*	*-9*	*-8*	*-7*	*-7*
Large-cap stocks	**-0.0**	**-0.5**	**-0.3**	**0.1**	**0.0**
t-statistic	*-0.1*	*-1.4*	*-0.6*	*0.1*	*0.1*
Mid-cap stocks	**-2.6**	**-3.9**	**-4.7**	**-4.6**	**-6.5**
t-statistic	*-7*	*-8*	*-8*	*-6*	*-6*
Small-cap stocks	**-5.3**	**-5.5**	**-5.8**	**-6.7**	**-8.5**
t-statistic	*-7*	*-6*	*-5*	*-5*	*-5*

Preferred

1. SIGMA X coefficient from shortfall regression, VWAP algorithm market orders greater than 1,000 shares, Aug to Dec 07.
2. Execution half-life for a parent order is the volume-weighted mean of the execution times of the child orders.

Source: Goldman Sachs Equity Execution Strategies

We also estimated a set of regressions by order size and got similar results:[10] strong SIGMA X benefit for orders in mid-cap and small-cap stocks, weak SIGMA X benefit for large-cap stocks and **the SIGMA X benefit increases with order size**.

We experimented with many different regressions.[11] In all cases, the results were similar to the ones summarized in Exhibits 3 and 4. The one exception is small orders less than 0.25 percent of average daily volume (~2,400 shares):[12] for small orders SIGMA X crossing does not reduce execution shortfall. We discuss this intriguing finding in the next section.

SIGMA X crossing and the reduction in information leakage

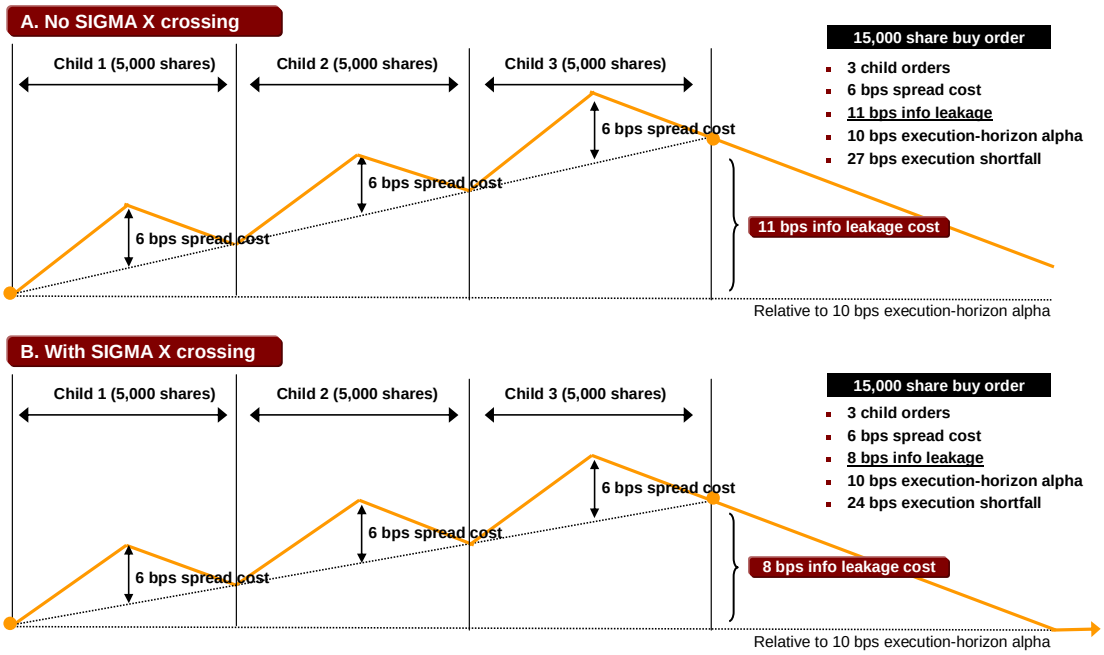
For small orders that execute fast, execution shortfall is dominated by the prevailing quoted spread and where the order executes within the spread (spread capture or price improvement). Crossing networks and the public market (exchanges, ECNs) all offer the possibility of some spread capture. Our finding that crossing does not reduce execution shortfall on small orders suggests that crossing does not increase spread capture relative to what is available in the public market.

The SIGMA X benefit that we observe on large orders, therefore, does not arise from increased spread capture at the child order level. Unlike small orders, larger orders execute over time and execution shortfall is sensitive to information leakage.[13] **Since the SIGMA X benefit we observe on large orders does not arise from increased spread capture at child order level, it most probably arises from reduced information leakage**.[14]

In Exhibit 5, we use a stylized example to illustrate one scenario of how SIGMA X crossing may reduce execution shortfall on large orders.[15] In the example the algorithm slices up a 15,000-share buy order into three 5,000-share child orders and executes them over time. In Case A there is no SIGMA X crossing and the execution shortfall on the 15,000 shares is 27 bps: 6 bps spread cost per child order, 10 bps execution-horizon alpha and 11 bps information leakage. In Case B the same order executes with 30 percent SIGMA X crossing. The spread

cost and alpha are the same as with no SIGMA X crossing, but the information leakage is reduced to 8 bps and the overall execution shortfall drops to 24 bps.

Exhibit 5. Reduced information leakage: stylized example[1]



1. Based on the characteristics of the average order in our preferred regressions sample.

Source: Goldman Sachs Equity Execution Strategies

SIGMA X crossing reduces information leakage because the Goldman Sachs Liquidity Enhanced VWAP algorithm places child orders in SIGMA X *without displaying them to the market*. If the child order executes within SIGMA X then it is never displayed to the market ahead of the trade. If the order does not execute within SIGMA X, the algorithm routes it to the public market where it is displayed.

Concluding comments

In this issue of Street Smart we confirmed the importance of taking full advantage of pools of non-displayed liquidity like SIGMA X when executing large orders. Our analysis shows that the execution quality of the Goldman Sachs VWAP algorithm improves with increased SIGMA X crossing. Building on this success, Goldman Sachs is changing the child order placement logic of all its algorithms to increase exposure to SIGMA X liquidity.

Our estimate of a 12 percent reduction in execution shortfall is based on the assumption of 30 percent SIGMA X crossing rate, the current average. But SIGMA X liquidity is growing rapidly; a year ago SIGMA X was crossing 30 million shares daily compared to 140 million shares currently. As SIGMA X liquidity continues to grow, the SIGMA X crossing rate will likely further increase and so, presumably, will the SIGMA X benefit. Based on our regression estimates, **a 50 percent SIGMA X crossing, for example, will reduce execution shortfall by 20 percent.**

One concern is that the proliferation of crossing networks keeps crossing rates at each venue relatively low. This proliferation, however, is a useful transitory phenomenon that encourages innovation.[16] We believe that market forces will inevitably lead to consolidation and the resulting increase in crossing rates will lead to even less information leakage and will further reduce execution shortfall.

APPENDIX: Details of the execution shortfall regressions

Exhibit A shows one of the many shortfall regressions we estimated. This is the preferred regression we use for the all-stock simulations in Exhibit 3. The sample consists of VWAP algorithm orders greater than 1,000 shares and with execution half-life greater than 30 minutes. We exclude orders with client-specified limit prices. The sample period is August through December, 07 and the final regression sample has 83,892 orders.

The explanatory power of our preferred regression (R-square) is a relatively high 71 percent. Most of the explanatory power comes from one factor: the execution-horizon (EH) alpha. We define EH-alpha as the price move from order arrival to the close, pro-rata allocated to the execution horizon.[17] In our regressions, the EH-alpha factor essentially removes the underlying price trend from the execution shortfall. EH-alpha has close to one-to-one effect on execution shortfall: an increase in EH-alpha of 10 bps increases execution shortfall by 11 bps.

Exhibit A. Preferred regression[1]

Dependent variable: execution shortfall (bps)		
	coefficient	*t-statistic*
Intercept	**-1.4**	*-1*
Participation rate[2]	**0.5**	*6*
Volatility over execution horizon	**0.04**	*17*
Bid-ask spread over execution horizon	**0.1**	*2*
Alpha over execution horizon	**1.1**	*438*
Mid-cap stocks	**2.1**	*3*
Small-cap stocks	**5.7**	*6*
NYSE stocks	**-4.5**	*-7*
Sell order	**7.5**	*12*
Cancel rate	**14.5**	*11*
SIGMA X crossing rate (φ)	**-10.6**	*-9*
Sample size	83,892	
R-square	**71%**	
Average execution shortfall	24 bps	

Factors (X) influencing execution shortfall

Most important factor

SIGMA X benefit: crossing reduces execution shortfall

1. VWAP algorithm market orders greater than 1000 shares and execution half-life greater than 30 minutes, August to December 07.
2. We use a non-linear Box Cox transformation of the participation rate.

Source: Goldman Sachs Equity Execution Strategies

In addition to EH-alpha our preferred regression has eight other factors:

- Participation rate defined as executed quantity divided by consolidated volume over the execution horizon. In our regressions we use a non-linear transformation of the participation rate. The participation rate measures the client-chosen execution aggressiveness. As expected, the participation rate has a positive sign in our regressions: all else equal, higher aggressiveness increases shortfall.
- Volatility over execution horizon. As expected, volatility has a positive sign in our regressions: all else equal, higher volatility increases shortfall.
- Bid-ask spread over execution horizon. Again as expected, the spread has a positive sign in our regressions: all else equal, higher spreads increase execution shortfall.
- Mid-cap indicator identifying orders in mid-cap stocks. The regression estimates suggest that, all else equal (including the participation rate), the average mid-cap order in our sample is 2.1 bps more expensive than the average large-cap order.
- Small-cap indicator identifying orders in small-cap stocks. The estimates suggest that, all else equal, the average small-cap order is 5.7 bps more expensive than the average large-cap order.
- NYSE indicator identifying orders in NYSE stocks. The estimates suggest that, all else equal, the average NYSE order is 4.5 bps cheaper than the average NASDAQ order.
- Sell order indicator. The estimates suggest that, all else equal, the average sell order is 7.5 bps more expensive than the average buy order.
- Cancel rate. A small number of orders in our sample are partially filled because the client cancelled them while executing. Since our sample excludes limit orders the main reason a client will cancel an order is when the execution is getting out of line with expectations. Our regression estimates support this hypothesis: partially filled cancelled orders, all else equal, are 14.5 bps more expensive than the average order.

All these factors are statistically significant (using a t-value of 2 as the cut-off).

After controlling for all other factors, our preferred regression in Exhibit A shows that the SIGMA X crossing rate has a coefficient of -10.6 bps and t-statistic of -9. There are several ways to interpret the SIGMA X coefficient. One is to assume that an order has 100 percent SIGMA X fill rate. In this case, our regression estimates suggest that the average order in our sample will be 10.6 bps cheaper if it executes totally within SIGMA X compared to zero SIGMA X execution. The average order in our sample, however, has a 30 percent fill rate. So a more realistic interpretation is to calculate the SIGMA X cost saving assuming a 30 percent fill rate. In this case, the average order in our sample with 30 percent SIGMA X crossing is 3.2 bps cheaper than the same average order without SIGMA X crossing.[18]

[1] SIGMA X consists of a client-to-client crossing network and several external liquidity providers. SIGMA X allows clients to take liquidity from non-displayed sources and benefit from the aggregated liquidity that flows through the firm's infrastructure.

[2] For each executed VWAP parent buy order, we define execution shortfall as the volume-weighted execution price minus the midquote at order arrival (strike price) as a percent of the strike price. For each executed sell order, shortfall is the strike price minus the volume-weighted execution price as a percent of strike price. We repeated the analysis for slippage to trade-life VWAP with similar results.

[3] The figures are for the first quarter of 2008 through March 25. The 140 million figure double counts the volume executed within the SIGMA X crossing network and single-counts the volume SIGMA X executes against external liquidity providers.

[4] Information leakage is not completely eliminated: crossing networks like SIGMA X do not display orders pre-trade but once the order is executed it will be reported to the Consolidated Tape.

[5] We exclude VWAP orders with limit prices because these orders have low fill rates complicating the analysis. We also exclude from our sample other unusual orders, for example, orders in low-priced stocks (less than $1), orders in high-priced stocks (greater than $150), orders in Pink Sheet stocks, ADRs, ETFs, etc.

[6] We measure the execution horizon by the order's execution half-life. For each executed parent order with multiple child orders we calculate execution half life as follows: we first calculate the execution time for each child order (arrival strike time to child execution time) and then calculate the value-weighted mean of each child order's execution time.

[7] In these simulations, we use the estimated regression coefficients and the mean values of the various factors (participation rate, volatility, spread, alpha, etc.) to generate the simulated shortfall.

[8] Large-cap stocks: market capitalization greater than $8 billion. Small-cap stocks: capitalization less than $1 billion.

[9] In all cases the SIGMA X benefit is statistically significant (t-statistic greater than two).

[10] We divided our sample into size buckets depending on order size as percent of average daily volume and re-estimated the regressions within each size bucket.

[11] For example, we estimated separate regressions for NYSE and NASDAQ stocks, we tried other possible factors and different functional forms. Our results were robust. We also estimated regressions for the January to July 07 period, before the introduction of the Liquidity Enhanced VWAP. Again we found a SIGMA X crossing benefit, but weaker than after the introduction of the Liquidity Enhanced VWAP.

[12] The median size for orders less than 0.25 percent of ADV is approximately 2,400 shares.

[13] For large orders that execute over time, execution shortfall is dominated by execution-horizon alpha, the underlying price move over the execution horizon (see Abrokwah & Sofianos, Shortfall Surprises, *Journal of Trading*, Summer 07). In our stylized example we assume information leakage causes an additional price move against the order and that this additional price move reverts. In practice, information leakage may also take the form of accelerating the incorporation of alpha in the stock price and may not revert.

[14] There is one more possibility: SIGMA X crossing slightly front loads the execution profile and if the order is associated with positive short-term alpha this front-loading will also reduce execution shortfall.

[15] The example is based on the characteristics of the average order in our preferred regressions sample: order size 16,000 shares, half-spread 5.5 bps, execution shortfall 26.8 bps without SIGMA X crossing and 23.6 bps with 30 percent crossing. We assume the execution horizon alpha (aside from information leakage) is 10 bps.

[16] For a discussion of this point, see Sofianos, Dark Pools and Algorithmic Trading, in *Algorithmic Trading, A Buy-Side Handbook (2nd Edition), The Trade*, July 07.

[17] See Abrokwah & Sofianos, Shortfall Surprises, *Journal of Trading*, Summer 07.

[18] These are the results we focus on in the main body of this report.

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